POSTDOCTORAL TRAINING PROGRAM
IN MATHEMATICAL THINKING, LEARNING, AND INSTRUCTION
APPLICATION R305B130007

THE FOLLOWING LISTS THE PROJECTS IN WHICH THE SENIOR MEMBERS OF THE PROPOSED TRAINING PROGRAM ARE INVOLVED. THIS INCLUDES (A) CURRENTLY ACTIVE PROJECTS, (B) PROJECTS UNDER REVIEW, AND (C) COMPLETED PROJECTS FOR WHICH DATA ARE AVAILABLE FOR ANALYSIS.CURRENTLY ACTIVE PROJECTS

1.a. National Center for Cognition and Math Instruction (NCCMI). Nathan (PI), Alibali (co-PI).

b. Brief description of the research questions and design and analytic methods: This national center—"the Math Center"—will bring together leading experts in cognition, instruction, assessment, research design and measurement, mathematics education, and teacher professional development around the core goal of redesigning components of a widely-used middle school mathematics curriculum—*Connected Mathematics Project* (CMP). The Math Center team will focus on applying four primary design principles from the IES Practice Guide, *Organizing instruction and study to improve student learning* (NCER 2007-2004) to the revision of mathematics curriculum: (1) combining graphics with verbal descriptions in ways that promote the integration of concepts, (2) structuring practice by interleaving worked samples with new problems to solve, (3) carefully spacing the learning of critical content and skills over time, and (4) using focused feedback on quizzes and homework to promote student learning. The primary focus on the Wisconsin campus will be on integration of visual and verbal information. The major studies entail a large randomized control trial (RCT) study, along with a number of smaller studies testing secondary research questions using experimental methods, think aloud reports, and eye tracking.

c. Setting, population, and intervention (if applicable): The Math Center will use established, evidence-based principles derived from experimental studies in classrooms and controlled laboratory settings. Our goal is to enhance the conditions of instruction and improve learning outcomes for students in important and challenging mathematics concepts and skills. Woven together with the redesign are large and small studies about how the design principles may contribute, individually and collectively, to substantial improvements in student learning and engagement. We will use a three-level hierarchical linear model to estimate intervention impacts at levels of student, teacher, and school.

Our initial, large, randomized control trial (RCT) study sample will include 78 middle schools randomly assigned to either experimental or control condition to assure a final sample of at least 35 schools in each cell. In each school, we expect the participation of all teachers who have at least 1 year of experience implementing the original curriculum (~2) and of all their students (three classes), resulting in an initial study sample of approximately 140 middle school teachers and 10,500 students. Measures include: student learning outcome measures (state assessments, and Mathematics Diagnostic Training Program for pre-algebra readiness assessment); student engagement measures; teacher knowledge measures; and fidelity of implementation measures.

The Center will also conduct an integrated series of design studies; develop and test practical guidelines that will enable mathematics teachers, curriculum developers, and publishers to

apply the design principles; conduct supplementary studies on important issues in math teaching and learning; disseminate findings; and provide leadership to the education field. Supplementary studies include student problems solving, think aloud reports, and eye tracking studies to assess attention, integration, and level of student engagement (pupillometry data).

d. Bullets for the IES topic area(s) and research goal(s) that the project most closely aligns with; the funding agency, amount, and performance period; and the types of roles fellows may have.

- IES Topic: CASL and Math-Science.
- IES Goal(s): 1, 2, 3, 5.
- Agency: IES-R&D Centers. Award: $9,998,406.00
- Performance Period: 9/1/10–8/31/15
- Postdoctoral Fellows Roles:
    - Large-scale randomized control trial efficacy study
    - Experimental design
    - Multilevel modeling using large-scale RCT data sets
    - Item design
    - Curriculum analysis
    - Item-level analysis
    - Protocol analysis of verbal and written solutions
    - Eye tracking

2a. **How do instructional gestures support students' mathematics learning? Alibali (PI), Nathan (Co-PI), and Knuth (Co-PI)**

b. Brief description of the research questions and design and analytic methods: In this project, we use experimental design across six experiments to focus on middle school (grades 6, 7, and 8) mathematics teachers' instructional gestures, with a specific focus on gestures that occur in instructional episodes that link related representations of mathematical information, in particular, gestures that serve to ground ideas in the physical environment or in familiar actions, experiences or representations. The research has three specific aims:

Aim 1. To investigate whether middle school mathematics students' learning is facilitated if the teacher grounds both (as compared to one or neither) of the to-be-linked ideas with gestures (Experiments 1 and 2);

Aim 2. To examine whether middle school mathematics student learning is facilitated if the teacher grounds the links using redundant rather than complementary gestures (Experiments 3 and 4); and

Aim 3. To examine whether gestures during middle school mathematics lessons offer a "special" way to ground ideas, in the sense that they are more effective at doing so than other, non-gestural methods of grounding (Experiments 5 and 6).

We will address each of these aims in two domains: *early algebra*, with a specific focus on the concepts of slope and intercept as they apply to linear functions; and *inferential statistics*, with a specific focus on confidence intervals and their connections to concepts of variability and distinctions between sample and population means.

c. Setting, population, and intervention (if applicable): Under each aim, two parallel experiments will be conducted: one in early algebra with seventh-grade participants, and one in statistics with college students. We plan to include 25 participants *per condition* in each of the proposed studies.

Seventh-grade participants will be recruited from the Madison Metropolitan School District (MMSD); the data collection will take place in the PIs' research space at the Wisconsin Center for Education Research. Because the middle schools in the MMSD all use the same curriculum, it will be straightforward to ensure that students have not already covered the target concepts (slope and intercept) before participating in the studies. The student body of middle school students in the MMSD is 51% Caucasian, 24% African American, 14% Hispanic, and 11% Asian (data from fall 2008).

Data for the studies that focus on statistics content will be collected in part at UW and in part at Northeastern Illinois University (NEIU). We will collect this data at both sites in order to achieve an ethnically and racially diverse sample. The undergraduate population at UW is not very racially or ethnically diverse (83% Caucasian, 3% African American, 3% Hispanic, and 6% Asian; 2007 data). The undergraduate population at NEIU is much more diverse (47% Caucasian, 10% African American, 25% Hispanic, and 10% Asian). We expect that drawing participants from both universities will also give us a broader range of student abilities, as admissions criteria at UW are more stringent than those at NEIU.To address Aim 1, we will conduct two experiments that involve video lessons that vary in whether neither, one, or both of the math representations to be linked are expressed in gesture as well as speech. In each experiment (one algebra, and one statistics) we will assess middle school students' conceptual and procedural knowledge of the target domain before and after the lessons, so that we can evaluate whether variations in the number of representations grounded in gesture influence students' learning.

To address Aim 2, we will conduct two experiments that involve video lessons that vary in whether linking episodes are presented using redundant speech and gestures or complementary speech and gestures. We will assess middle school students' algebra and statistics knowledge before and after the lessons, to evaluate whether variations in the teacher's gesture-speech relationship influence students' learning.

To address Aim 3, we will conduct two experiments that involve beginning algebra and statistics lessons presented with PowerPoint. In each experiment, one lesson will show the teacher using gestures to ground the to-be-linked representations that are presented on slides; in the other, the to-be-linked representations will be highlighted using graphics and animation, such as color and blinking. We will assess students' knowledge before and after the lessons, to evaluate whether variations in the method of grounding influence students' learning. Thus, we will ascertain whether grounding with gestures is more effective than other methods of grounding.

d. Bullets for the IES topic area(s) and research goal(s) that the project most closely aligns with; the funding agency, amount, and performance period; and the types of roles fellows may have.

- IES Topic: Math-Science (In NSF: "Research on Emerging Topics in STEM Education: Cognitive Processes Underlying STEM Learning and Teaching.")
- IES Goal(s): 1, 2.
- Agency & Amount: NSF REESE; $999,789
- Performance Period: Sep 2009–Aug 2014
- Roles for Fellows:
    - Data collection and entry
    - Experimental data analysis using regression and multilevel models
    - Gesture analysis from video
    - Video production
    - Student strategy and error analysis

3.a. Supporting Students Proof Practices through Quantitative Reasoning in Algebra (SPARQ). Ellis (PI).

b. Brief description of the research questions and design and analytic methods: This project addresses the challenge of proof instruction in secondary mathematics classrooms by studying the ways in which quantitative reasoning acts as a foundation for fostering students' proof skills in algebra. This research occurs in four major phases, with data collection and analysis for each phase building on the previous phase's work. Phase 1 entails small-scale teaching experiments on linear, quadratic, and exponential functions aimed at the formulation of evidence-based learning trajectories of middle-school students' emerging function understanding and proof competencies. Phase 2 replicates the teaching-experiment units with larger student groups in order to test the validity of the learning trajectories. Phase 3 incorporates findings from Phases 1 and 2 to implement a teaching professional development program for in-service middle school teachers. Phase 4 scales up the project results in whole-classroom implementation of replacement units. The analytic method for each phase draws on the constant comparative method used in the development of grounded theory (Strauss & Corbin, 1990). In Phases 1 and 2, we use open and axial coding techniques to formulate and validate learning trajectories and students' proof schemes (Harel, 2007). In Phases 3 and 4 we rely on cross-case analysis (Munger & Psencik, 2002) methods to identify teachers' conceptual trajectories and patterns of lesson implementation. Written student assessment data at the completion of each whole-classroom unit will identify students' function understanding and proof competencies.

c. Setting, population, and intervention (if applicable): Phases 1 and 2 occur at a public middle school with seventh and eighth grade pre-algebra and algebra students. Each small-scale teaching experiment will recruit eight participants and implement a 15-day unit for 1 hour each day; all sessions are videotaped and transcribed.

The participants for Phase 3 are 20 in-service middle school teachers who participate in a semester-long, 3-credit professional development course on algebraic reasoning and proof. The course consist of a 3-hour weekly session over a 15-week period. Each session is observed and videotaped, and all artifacts from the course are collected and analyzed.

Phase 4 consists of a subset of six teachers from Phase 3 and the students in each of these teachers' middle school classrooms (two seventh-grade teachers and four eighth-grade teachers). Each teacher will implement replacement units for linear functions (seventh grade) or quadratic and exponential functions (eighth grade). The intervention consists of the linear, quadratic, and exponential functions replacement units developed from Phases 1 and 2 and implemented in place of the standard instructional units currently in use. Data collection will include videotaped classroom observations during the entirety of the replacement units, students' written work, and written assessment data on students' function understanding and argumentation strategies at the completion of each unit.

d. Bullets for the IES topic area(s) and research goal(s) that the project most closely aligns with; the funding agency, amount, and performance period; and the types of roles fellows may have.

- IES Topic: Math-Science
- IES Goal(s): 1, 2
- Agency: NSF CAREER program through DRK–12. Award: $730,417.
- Performance Period: 9/1/10–8/31/15)
- Postdoctoral Fellows Roles:
    - Teaching experiment, professional development, and whole-class scale-up study
    - Teaching experiment task design

- Adaptation and revision of lesson activities
- Teaching-experiment data collection activities, including instruction
- Data coding, student activity analysis, and learning trajectory development
- Professional development implementation
- Classroom observation data collection
- Written assessment analysis

4.a. Professional development for culturally relevant teaching and learning in Pre–K mathematics. Mentor: Wager (PI)

b. Brief description of the research questions and design and analytic methods: This project addresses the challenge of providing equitable opportunities for young children to learn mathematics. Through the project we have designed professional development to support pre–K teachers to provide culturally and developmentally responsive mathematics pedagogy. Through the professional development, teachers are learning to identify mathematics children engage with during play, and identify and build on mathematical practices from children's homes with the goal of assuring that all children engage in rich mathematics learning. The project is currently in the third of 4 years and we are working with our third and final cohort of classroom teachers. The research questions include:

- How does a professional development program for culturally relevant teaching and learning in pre–K mathematics help teachers recognize the mathematical resources available in their students' homes, and how do teachers use these home resources to deepen their instructional practices?
- How is teachers' knowledge of pre–K students' mathematical thinking and learning opportunities related to student achievement?
- How successful is this professional development in reducing the achievement gap?

The first question has been (and continues to be) explored through ethnographic and narrative methodologies. The second and third questions will be explored through statistical analysis of classroom outcomes. We plan to analyze disaggregated grades and assessment scores of children whose teacher participated in the professional development with those who did not.

c. Setting, population, and intervention (if applicable): The professional development has been provided to teachers of a new public 4-year-old kindergarten program. The students in the teachers classrooms come from diverse economic, ethnic, and linguistic families. A total of 35 teachers participated in 2 years of the professional development and approximately 1,500 children in their classrooms.

d. Bullets for the IES topic area(s) and research goal(s) that the project most closely aligns with; the funding agency, amount, and performance period; and the types of roles fellows may have.

- IES Topic: e.g. *Math-Science*
- IES Goal(s): 1, *2*.
- Agency: NSF DRK–12). Award: $2,180,326.
- Performance Period: 9/1/10–8/31/14)
- Postdoctoral Fellows Roles:
    - Analysis of student outcomes data to measure effectiveness of professional development
    - Classroom observations
    - Student interviews
    - Data coding and analysis

- Developing scale up opportunities for the professional development

5.a. Matching strategies for observational studies with multilevel data in educational research. Steiner (PI), J-S Kim (co-PI)

b. Brief description of the research questions and design and analytic methods: The project seeks to develop, test, and disseminate matching strategies for causal inference with observational multilevel data in educational research. Matching strategies for non-equivalent control group designs with multilevel data are required since randomized experiments cannot always be conducted—due to ethical reasons, for example. The project investigates different within- and across-cluster matching strategies for estimating causal treatment effects across and within clusters. In using theoretical investigations, simulation studies, within-study comparisons, and analyses of Early Childhood Longitudinal Study, Kindergarten Class of 2010–11 data, two main goals are pursued. First, determine under which conditions within- and across-cluster matching strategies produce consistent estimates of average treatment effects. Second, find which matching strategies and analytic approaches work best in educational research practice. In evaluating the matching strategies, different propensity score estimation methods (fixed vs. random effects models) and various matching techniques including propensity score matching, inverse-propensity matching, propensity score stratification, or Mahalanobis distance matching are investigated.

c. Setting, population, and intervention (if applicable): not applicable.

d. Bullets for the IES topic area(s) and research goal(s) that the project most closely aligns with; the funding agency, amount, and performance period; and the types of roles fellows may have.

- IES Topic: Statistical and Research Methodology in Education
- IES Goals: N/A
- Agency: IES. Award: $588,000.
- Performance Period: 01/07/2012–30/06/2015
- Postdoctoral Fellows Roles:
  - Quasi-experimentation/ Matching designs
  - Propensity score analysis
  - Multilevel modeling
  - Within-study comparisons

6.a. The role and use of examples in learning to prove (EXAMPLES). Knuth (PI), Ellis (co-PI)

b. Brief description of the research questions and design and analytic methods: The goals of the project are to (a) investigate the nature of middle school and high school (*secondary school*) students', undergraduate students', and mathematicians' thinking about the examples they use when developing, exploring, and proving conjectures; (b) investigate ways in which thinking about and analyzing examples may facilitate the development of students' learning to prove; and (c) develop instructional materials designed to help teachers and university instructors foster the development of their students' thinking about and use of examples in learning to prove.

The research will occur in two major phases and includes four participant groups: Phase 1, interview studies of secondary school students, undergraduate mathematics majors, and mathematicians; Phase 2, small-group teaching experiments with secondary school students and undergraduates.

c. Setting, population, and intervention (if applicable): The research settings include middle schools, high schools, and universities; subjects include middle and high school students, undergraduate mathematics majors, and university mathematicians. Each interview study will be conducted with approximately 15 subjects from each subject population, and each 15-week teaching experiment will be conducted with approximately 12–15 subjects from each of the three populations (middle school, high school, and undergraduates).

d. Bullets for the IES topic area(s) and research goal(s) that the project most closely aligns with; the funding agency, amount, and performance period; and the types of roles fellows may have.

- IES Topic: CASL, MSE
- IES Goals: 1, 2
- Agency: NSF DRK–12 Program. Award: $995,997.
- Performance Period: 9/2012–8/2015
- Postdoctoral Fellows Roles:
    - Instrumentation design
    - Conducting interviews
    - Teaching experiment design
    - Teaching experiment implementation
    - Data analyses
    - Development of instructional materials
    - Dissemination of results

7.a. The impact of early algebra on students' algebra readiness (IMPACT). Knuth (PI)

b. Brief description of the research questions and design and analytic methods: This project builds on the DRK–12 project *Developing algebra-ready students for middle school: Exploring the impact of early algebra*. The goals of the project described here are twofold: (1) Conduct a longitudinal study to measure the impact of a comprehensive, sustained early algebra intervention in grades 3–5 on students' algebra understanding in elementary grades and their algebra readiness in middle school; and (2) determine the effectiveness of the early algebra intervention in naturalistic settings. The project uses a quasi-experimental design to compare the performance of students who receive an early algebra intervention to students who receive more traditional elementary grades instruction.

c. Setting, population, and intervention (if applicable): Approximately 450 students from two elementary schools (one treatment school and one control school) are participating in the research. The intervention is based on the early algebra curricular learning progression developed in the prior project. In addition, the previously developed and validated grade-based assessments will be used to measure student performance. A member of the project team is serving as the instructor for all of the treatment classes.

d. Bullets for the IES topic area(s) and research goal(s) that the project most closely aligns with; the funding agency, amount, and performance period; and the types of roles fellows may have.

- IES Topic: MSE
- IES Goals: 3
- Agency: NSF DRK–12 Program. Award: $939,935.
- Performance Period: 9/2012–8/2015)
- Postdoctoral Fellows Roles:
    - Experimental design

- Assessment data analyses (both quantitative and qualitative)
- Development of professional development materials
- Dissemination of results

8.a. Connecting mathematical ideas through animated multimodal instruction. Alibali (PI), Nathan (Co-PI)

b. Brief description of the research questions and design and analytic methods: In this Cognition and Student Learning Goal 1 (Exploration) proposal, we use experimental design to study the effects of instruction that makes connections between mathematical ideas in the domain of middle school algebra.

Linking episodes—defined as segments of instruction in which teachers connect ideas—are integral to mathematics instruction and an important contributor to students' learning. In the proposed work, we will study how instruction that includes linking episodes affects middle school mathematics learning using a novel tool: GALE (Gesturing Avatar for Learning and Education), a software-based system that involves a human-like avatar teacher who can speak, gesture, and write. We will use this system as a test bed for understanding which ways of communicating about connections among ideas are most effective for fostering student learning in mathematics.

We will use avatar-based instruction in experimental studies with middle student participants to test hypotheses about how to most effectively link ideas in algebra instruction. We will test several factors that may influence the effectiveness of teachers' communication about links between ideas: (1) whether links are expressed in speech with linking gestures, gestures that do not link, or no gestures, (2) whether gestured links highlight element-by-element correspondences or more general relationships, and (3) whether links between ideas are expressed using simultaneous or sequential gestures. Once we identify the most effective ways of linking ideas in instruction, we will compare avatar-based linking instruction to instruction provided by human teachers. Finally, we will explore whether the avatar can be used to help human math teachers improve their own communication about links between mathematical ideas.

The proposed work has three specific aims:

Aim 1. Adapt an existing software system/avatar to serve as a test bed for hypotheses about linking in early algebra instruction;

Aim 2. Address four specific questions about how to most effectively use gestures and speech to link ideas in early algebra instruction:

(1) Do lessons in which the instructor uses both speech and gestures to link representations lead to greater learning than lessons that use speech alone or speech with gestures that do not link representations?

(2) Do lessons in which the instructor delineates correspondences between related representations using *element-by-element* linking gestures lead to greater learning than lessons in which the instructor uses more general linking gestures?

(3) Do lessons in which the instructor uses *sequential* gestures to corresponding aspects of related representations lead to greater learning than lessons in which the instructor uses simultaneous gestures?

(4) Are lessons *delivered via avatar* more effective, as effective, or less effective than comparable lessons delivered by a video of a human teacher?

Aim 3. Test whether the avatar-based linking instruction can be used as a tool to help teachers improve their algebra instruction.

c. Setting, population, and intervention (if applicable): *Participants*. For Experiments 1a–4a (linear functions), participants will be 40 sixth- or rising seventh-grade students per condition, and for Experiments 1b–4b (polynomial multiplication) participants will be 40 seventh- or rising eighth-grade students per condition. Data for Aims 2 and 3 will be collected in Madison, Wisconsin and Iowa City, Iowa. Student participants at the Madison site will be recruited from MMSD. The data collection will take place in the PIs' research space at the Wisconsin Center for Education Research. Because the middle schools in the MMSD all use the same curriculum, it will be straightforward to ensure that students have not already covered the target concepts before participating in the studies. The student body of middle-school students in the MMSD is 46% Caucasian, 23% African American, 16% Hispanic, 9% Asian, 1% Native American, and 6% multiracial.

We plan to include at least 40 student participants *per condition* in each study under Aim 2. Student participants at the Iowa site will be recruited using a database of child participants that is maintained by the Psychology Department at the University of Iowa (UI). At present there are approximately 2,500 children in the database who will be the appropriate age for this project in the time period proposed for data collection. We will use the pretest data to ensure that students have not already covered the target concepts. A large proportion of participants in the UI database attend the Iowa City Community Schools; the student body in this district is 67% Caucasian, 16% African American, 9% Hispanic, and 7% Asian; therefore, we anticipate being able to recruit a diverse sample.

d. Bullets for the IES topic area(s) and research goal(s) that the project most closely aligns with; the funding agency, amount, and performance period; and the types of roles fellows may have.

- IES Topic: Cognition and Student Learning (CASL)
- IES Goal(s): 1
- Agency: IES
- Performance Period: June 2013–May 2017 (proposed)
- Roles for Fellows include:
    - Test construction
    - Student data collection
    - Data analysis using multilevel logistic models
    - Analysis of student think aloud reports
    - Manuscript preparation
    - Teacher professional development

9.a. Promoting discriminative and generative learning: transfer in arithmetic problem solving. Kalish (PI), Alibali (co-PI).

b. Brief description of the research questions and design and analytic methods: We will test an account of memory models, representations that students form as they practice solving problems; specifically, we will test malleable factors of practice that may lead students to form different kinds of memory models, and study tradeoffs between fluency and generality in learning. The study involves small-scale experiments testing the effects of different forms of practice solving mathematics problems. These lab-based experiments will inform the design of a classroom intervention, comparing different forms of practice. Data are analyzed using

descriptive and inferential statistics, both to test hypotheses and to compare student performance to model predictions.

c. Setting, population, and intervention (if applicable): The research will be conducted in a laboratory setting with samples of preschool- and young school-aged children recruited from the community. A classroom intervention is planned with second grade students in mathematics classrooms in a public elementary school. The intervention consists of three different 1-week practice activities designed to supplement existing mathematics instruction.

d. Bullets for the IES topic area(s) and research goal(s) that the project most closely aligns with; the funding agency, amount, and performance period; and the types of roles fellows may have.

- IES Topic: CASL
- IES Goal(s): 1, 5
- Agency: IES/CASL
- Performance Period: 6/1/13–5/31/17
- Postdoctoral Fellows Roles:
    - Experimental design
    - Cognitive modeling
    - Statistical data analysis

PENDING PROJECTS (UNDER REVIEW)

1.a. Project Title & Mentors: The integration of STEM knowledge across classroom contexts. Nathan (PI), Alibali (Co-PI)

b. Brief description of the research questions and design and analytic methods: We identified two important malleable factors in the form of communicative practices that teachers use to foreground the linking of STEM content within classroom contexts. The first malleable factor is projection, which is connecting the current STEM-content-focused classroom talk to past and/or future contexts. The second malleable factor is coordination, which is the linking of separate concepts, tools, or representations that are co-present (often juxtaposed) within the instructional space.

We propose to investigate the roles of projection and coordination (hereafter, P&C) in STEM integration through research addressing two related aims. Aim 1 is to document how communication practices such as P&C are used to link STEM-related language, concepts, and procedures within and across classroom contexts. Aim 2 is to assess the effectiveness of P&C on a variety of STEM-related learning outcomes, in particular, the ability of students to transfer their STEM knowledge to novel problem-solving contexts.

c. Setting, population, and intervention (if applicable): The sites of the ethnographic research will be high schools in Tuscaloosa, Alabama, and the surrounding area. The city of Tuscaloosa has approximately 100,000 people. Tuscaloosa City Schools (TCS) has 3234 students enrolled in its four high schools, 343 of which are enrolled at its STEM-focused high school, the Tuscaloosa Center for Technology (TCT): 65% of the students in Tuscaloosa City Schools received free or reduced-price lunch (a common poverty proxy), which is considerably higher than the average for Alabama (47.7%) and for the nation (39%). Data is not available for the TCT high school, but the three feeder schools reflect the high poverty proxy for the district

overall, at a combined average of 53.1%. The school catchment for the district is 44.2% African American and 50% Caucasian.

The data collected in the ethnographic study (Study 1) will provide rich evidence for the effects of projection and coordination on students' abilities to make links across classroom and disciplinary contexts. Experimental participants will include high school students (grades 9–12) recruited from high schools in Madison, Wisconsin; the experimental sessions will take place in the investigators' laboratory on the University of Wisconsin campus. The student body of middle-school students in the MMSD is 46% Caucasian, 23% African American, 16% Hispanic, 9% Asian, 1% Native American, and 6% multiracial. We will include a minimum of 40 participants per condition in each of the proposed experiments.

To address Aim 1, we will carry out an ethnography in STEM classrooms that will include four forms of data: (1) observations and field notes of classroom interaction, (2) interviews with the teachers and selected students on their goals and ideas about STEM knowledge and education, (3) collection of texts and artifacts produced during instruction, (e.g., textbooks, handouts, student portfolios), and (4) video recordings of teacher-student and student peer group interactions during classroom lessons. Ethnographic and video data will be collected from science, math, and technology/engineering classrooms in three STEM-focused high schools in Tuscaloosa, Alabama, and the surrounding region (totaling 9 classrooms). The ethnographic materials will inform interpretation of the video data, which will be subject to a discourse analysis, involving iterations of transcription and annotation, identifying and coding instances of P&C and related behaviors, and identifying patterns in the corpus. The analysis will yield a rich description of the roles of P&C in STEM integration in classrooms, which will inform work on Aim 2, which is to assess the effectiveness of P&C on STEM learning outcomes.

Experiments 2 & 3 will address Aim 2 by experimentally testing the effectiveness of P&C using pretest/posttest designs that implement variations in coordination (Experiment 2) and projection (Experiment 3) in lessons about the application of DeMorgan's Theorem to problems in digital electronics. Participants in Experiment 2 will be randomly assigned to one of three conditions that vary in the teacher's use of coordination: multimodal coordination, verbal coordination, or no explicit coordination. Participants in Experiment 3 will be randomly assigned to one of four conditions that vary in the teacher's use of projection forward in time and projection backward in time, implemented in a 2x2 factorial design.

The pretests and posttests will be used to assess student understanding of links, by asking students and to solve novel and complex problems and to translate among representations. Data will be analyzed using ANCOVA (controlling for pretest performance) and also with qualitative analyses of strategies, errors, and problem explanations. Thus, the experiments assess how variations in projection and coordination in the lessons foster student understanding of STEM content.

d. Bullets for the IES topic area(s) and research goal(s) that the project most closely aligns with; the funding agency, amount, and performance period; and the types of roles fellows may have.

- IES Topic: Cognition and Student Learning
- IES Goal(s): 1
- Agency: IES CASL
- Performance Period: June 2013–May 2017
- Roles for Fellows include:
    - Student and site recruitment
    - Ethnographic methods and data analysis
    - Classroom observations

- Experimental design
- Regression analysis
- Data entry and coding
- Interviews
- Writing and presenting work at scholarly meetings

2.a. Project Title & Mentors: Making middle school mathematics accessible through engineering enrichment. Nathan (PI)

b. Brief description of the research questions and design and analytic methods: This project uses clinical, classroom, curriculum, and professional development methods to produce cognitively-based learning tools and instructional strategies for proportional reasoning in middle school mathematics that can improve existing curricular and instructional practices. This is a collaboration between WestEd and the University of Wisconsin–Madison, which have a history of collaboration on interdisciplinary research in STEM education. In addition to the *content* focus, students are also meant to develop mathematical *practices,* including reasoning abstractly and quantitatively, constructing viable arguments and critiquing the reasoning of others, attending to precision, and modeling with mathematics.

The goal is supporting and examining student learning for constructing a rich network of connections and triggers so that mathematical and other knowledge comes to mind in the moment when required. In particular, the team will focus on student cognition and learning about proportional reasoning with surface area and volume when two design principles are applied to changes in the *Connected Mathematics Project* (CMP): (1) combining visual representations with verbal descriptions in ways that promote the integration of concepts, and (2) helping students build explanations by asking and answering deep questions. The IES-funded *National Center for Cognition and Mathematics Instruction* (including Wisconsin and WestEd) has already redesigned the *CMP* unit, *Comparing and Scaling*, using Principle 1. The project augments and extends that redesigned CMP unit and then researches student learning. We address Principle 2 with a supplement of deep-question materials and use both principles in creating an alternate Unit Project through revision of an engineering activity called *Solar Structures* (from IEEE TryEngineering). The same cognitive principles are used in creating associated professional development experiences for teachers. The project will produce a fully developed version of the proposed intervention, including prototypes of all materials and products necessary for implementation of the intervention in authentic school settings.

The project uses an iterative design cycle of (a) clinical and classroom qualitative and quantitative research, (b) curriculum development, and (c) teacher professional development and feedback. Our theory of change addresses how middle school mathematics students develop conceptual connections for reasoning with ratio and proportion. The project culminates with an experimental study in 50 classrooms (1200 to 1500 students).

The project is designed to meet five objectives that cover the intended, enacted, and achieved curriculum:

1. Apply the cognitive principles of visual-verbal integration and asking/answering deep questions to modify a curricular activity (the *intended* curriculum);
2. Conduct clinical (in lab) and design (in schools) studies on student cognition and learning during engineering-rich mathematics education with the revised materials (the *achieved* curriculum);
3. Apply lessons learned from 1 and 2, along with the cognitive principles, to create teacher professional development sessions for using the activity (the *enacted* curriculum);

4. Conduct Year 3 pilot study research on the effects of the specific curricular changes and teacher professional development on student achievement, engagement, and motivation (assess the *achieved* curriculum); and,

Actively disseminate products, models, research tools, and other results through publication, presentation, and technical assistance. Specially designed or selected measures for the study include: pre-post measures of student learning in the content domain; posttest measures of student attitudes/motivation; teacher surveys, electronic teacher logs in which teachers will report on their use of the curriculum activities, and observations of implementation. Data will be collected through electronic logs and survey data from teachers, and through observations of 10 randomly selected classrooms (5 from each treatment). The surveys will document class and homework time spent on materials, extent of implementation of the key elements of the revised instructional materials, teachers' use of core cognitive principles, and the frequency and types of classroom interaction.

c. Setting, population, and intervention (if applicable);

Several school districts that use CMP in Wisconsin, North Carolina, and Pennsylvania have agreed to serve as field test sites. These sites represent a range of size and demographics. These districts and schools have large proportions of students from populations that are traditionally underserved and associated with lower mathematics achievement (e.g., students who qualify for free or reduced-price school lunch, English language learners, African American, Hispanic, and Native American students).

We anticipate involvement of 6 to 10 teachers and their (approximately) 300 students in Year 1 clinical and design studies. Year 2 re-development and classroom testing of materials will involve at least 10 teachers (up to 20) and their students (300 to 600). Additionally, we expect Year 2 clinical work to involve about 120 students. In Year 3, we conduct an experimental comparison of student outcomes with 50 classes (25 in each condition; total of at least 1000, up to 1500 students).

d. Bullets for the IES topic area(s) and research goal(s) that the project most closely aligns with; the funding agency, amount, and performance period; and the types of roles fellows may have.

- IES Topic: Cognition and Student Learning
- IES Goal(s): 2
- Agency: IES CASL
- Performance Period: July 2013–June 2016 (proposed)
- Roles for Fellows include:
    - Professional development
    - Curriculum materials development
    - Test development
    - Working with State standards-based assessment data sets
    - Multilevel modeling
    - Student outcome measures and data analysis
    - Teacher knowledge measures and data analysis
    - Classroom observations of instruction
    - Implementation fidelity measures and data analysis

3. a. Project Title & Mentors: A comparison of early algebra instruction and arithmetic-based instruction on students' algebra-readiness for middle school. Mentor: Eric Knuth (PI)

b. Brief description of the research questions and design and analytic methods:

This project aims to (1) establish the effectiveness of an early algebra intervention, in demographically diverse settings, on children's algebra readiness, and (2) assess the fidelity with which elementary teachers implement the intervention in authentic, intact classrooms. The proposed work uses a cluster randomized trial research design, where treatment is implemented at the class level, to test the effectiveness of our early algebra intervention as participants progress from grades 3–6.

We will use hierarchical linear modeling to measure effects on students' performance and to account for variations in the effects, thus assessing the intervention's impact. Analysis will include multiple regression analyses designed to measure the relative contributions (e.g., time, math proficiency) on students' algebra-readiness. Analyses of variance will also be conducted to determine differences in algebra-readiness as a function of the elementary school regular mathematics curriculum and early algebra instruction. We will use quantitative and qualitative analyses of teacher data. Due to the nested nature of the data, we will use hierarchical linear modeling to test the hypothesis that teacher fidelity of implementation will positively predict student performance.

c. Setting, population, and intervention (if applicable);

The setting for this study includes five elementary schools (24 classrooms in grade 3, with participating students tracked from grades 3–6). The sample consists of approximately 624 students and 80 teachers in participating schools that represent racially and socioeconomically diverse communities (with school populations ranging from 20% to 74% low SES and 8% to 89% minority). The early algebra intervention was developed as part of a DRK–12 project: *Developing algebra-ready students for middle school: Exploring the impact of early algebra*. The intervention will be implemented with full grade-3 cohorts of students and will continue with these students as they progress through grade 5, with follow-up assessment in grade 6. Students in the control condition will be students in participating elementary schools, and during the early algebra intervention, they will receive "business-as-usual" instruction in mathematics (that is, their regularly planned instruction). Regular classroom teachers (i.e., not a member of the project team) will be the instructors in both conditions.

d. Bullets for the IES topic area(s) and research goal(s) that the project most closely aligns with; the funding agency, amount, and performance period; and the types of roles fellows may have.

- IES Topic: MSE
- IES Goal(s): 3
- Agency: IES
- Performance Period: 2013–2017
- Postdoctoral Fellows Roles:
    - Experimental design
    - Assessment data analyses (both quantitative and qualitative)
    - Classroom observation protocol development
    - Analysis of classroom data
    - Dissemination of results

COMPLETED PROJECTS FOR WHICH DATA ARE AVAILABLE FOR ANALYSIS

1.a. Project Title & Mentors: Tangibility for the teaching, learning, and communicating of mathematics. Nathan (PI), Alibali (co-PI).

b. Brief description of the research questions and design and analytic methods:

RQ1. ***Grounding*****.** What does it mean to ground something so that novel ideas and formal (abstract) representations take on meaning for participants in mathematical discourse?

We hypothesize that careful analysis of observational studies of the multimodal communication (gestures, posture, writing/drawing, object use) that happens during mathematical activity (among professionals, mathematicians, teachers and students) and classroom learning (in academic and vocational ed settings) will reveal how learners and workers ground new ideas and representations while functioning in natural settings.

Research methods include: (a) quantitative content analysis, and qualitative discourse analysis approaches to observations (multicamera video analysis); (b) analysis of multimodal behavior (speech, body movement, use of symbolic and depictive inscriptions) in high school geometry, and high school vocational education settings; and (c) collaborative data sessions.

RQ2. ***Action***. What is the role of action in learning geometry, and can an understanding of this role yield insights into why some aspects of geometry are harder to learn than others?

We hypothesize that as you move further away from (literal) action, there are fewer cognitive resources (attention, memory, meaning making) recruited.

Thus, we ask: Does mathematics that is highly action-based lead to advantages in learning, communication and performance over mathematics that is low-action?

Data are collected through laboratory and pull-out (tutorial) sessions. Research methods include: (a) experimental design; (b) analysis of learning (pre-post) and successful performance; and, (c) video analysis of gestures and simulated action during explanations of performance

c. Setting, population, and intervention (if applicable): Classrooms are drawn from Milwaukee Public Schools (MPS) and (MMSD). The MMSD student body at the time of data collection was: 46% Caucasian, 23% African American, 16% Hispanic, 9% Asian, 1% Native American, and 6% multiracial. MPS students at the time of data collection were: 57% of students were listed as African American, 22% Hispanics, 12% White, 4% Asian, and 4% Other. Approximately 72% of the students in the district came from low-income families who were eligible for free or reduced-price lunch. Students identified for special education services made up 18% of the student body, greater than the national average (12%), while English Language Learners made up 18% of the student body for the specific school sites where we conducted the research.

We seek a deeper understanding of the role of action in geometry and spatial reasoning more broadly, guided by the second research question that asks about the role of action in learning. We specifically ask whether an understanding of this role can yield insights into why some aspects of geometry are harder to learn than others. To address this question, we will carry out two experiments, one focused on academic geometry, and one on geometric and spatial reasoning in pre-engineering. Both experiments will investigate the role of action, by comparing learning and performance in high-action and low-action contexts. We hypothesize that tasks that explicitly call for action (e.g., proof by construction, debugging for logic circuit reliability) facilitate both the understanding of the justification process and the mathematical concepts being proved, while those that emphasize static relations over actions (e.g., proof by deduction, logic analysis

using propositions and truth tables) result in impoverished understanding of the proof process and the concepts being proved.

Each experiment will utilize a simple two-group, within-subjects design (N = 30 participants), in treatment and control conditions. In both experiments, we will examine amount of learning (using a pretest/posttest design) and transfer, as well as conduct qualitative analyses of participants' multimodal communication about the tasks (i.e., uses of actions, simulated actions, and inscriptions in defending their reasoning).

d. Bullets for the IES topic area(s) and research goal(s) that the project most closely aligns with; the funding agency, amount, and performance period; and the types of roles fellows may have.

- IES Topic: NSF-REESE version of Cognition and Student Learning
- IES Goal(s): 1
- Agency: NSF REESE
- Performance Period: Sep 2008–Aug 2013
- Roles for Fellows include:
    - Video coding analysis of collaborative learning classroom instruction
    - Analysis of student discourse
    - Experimental data analysis
    - Analysis of instruction and curriculum for STEM integration
    - Gesture analysis
    - Dissemination through scholarly writing and presentations

2.a. Project Title & Mentors: Understanding and cultivating the development of students' competencies in justifying and proving (PROOF). Knuth (PI)

b. Brief description of the research questions and design and analytic methods: The purpose of this study was to trace the development of middle school (grades 6–8) students' competencies in justifying and proving and to identify conditions that influence that development. This study followed a group of grade 6 students through the completion of grade 8, using written assessments and semi-structured interviews to document and analyze changes in their competencies. The objectives of this project were threefold: (1) to understand the development of students' competencies in justifying and proving; (2) to understand the conditions and pedagogy necessary to promote the development of those competencies; and (3) to develop teacher preparation and professional development materials designed both to enhance teachers' understandings of proof and to support them in fostering the development of students' competencies in justifying and proving.

c. Setting, population, and intervention (if applicable): The research setting included a local middle school. Approximately 100 grade 6 students were followed through the completion of grade 8, and written assessments were administered at four time points (beginning of grade 6, beginning of grade 7, beginning and end of grade 8). Interviews were conducted each year. In addition, each year of the longitudinal study, assessment data were collected from the entire middle school (approximately 350 students, includes longitudinal study students).

d. Bullets for the IES topic area(s) and research goal(s) that the project most closely aligns with; the funding agency, amount, and performance period; and the types of roles fellows may have.

- IES Topic: CASL, MSE
- IES Goal(s): 1
- Funded by: NSF CAREER Program. Award: $701,649.

- Performance Period: 9/2001–8/2006.
- Postdoctoral Fellows Roles:
    - Analyses of assessment and interview data
    - Dissemination of results

3.a. Project Title & Mentors: Coordinating social and individual aspects of generalizing activity: A multi-tiered "focusing phenomena" study. Ellis (Co-PI) (with PI J. Lobato)

b. Brief description of the research questions and design and analytic methods: A central and enduring goal of education has been to provide learning experiences that generalize beyond the specific conditions of initial learning. However, research studies and national assessments in mathematics indicate that students often perform poorly on real world applications, and many students graduate unable to connect school mathematics to work or everyday settings. Furthermore, researchers' progress in supporting the generalization of learning has been limited due to theoretical problems with the transfer construct. One reason why attempts to help students productively generalize their learning experiences have not been as successful as anticipated may be because existing accounts do not adequately account for both social and individual aspects of generalizing activity. The construct of *focusing phenomena* was developed to account for the ways in which features of social environments influence what students attend to mathematically; *focusing phenomena* are observable features of classroom environments that regularly direct students' attention toward certain (mathematical) properties or regularities. Exploratory comparative work suggests that it is possible to generate a contrasting set of focusing phenomena (across a variety of instructional treatments) linked conceptually with an associated set of students' generalizations for a given topic. In the proposed study, profiles are developed in a more rigorous and systematic way, developing predictive models that describe how changing the nature of focusing phenomena affects the nature of the associated individual generalizations. To accomplish this goal, a research design comprised of a set of integrated studies is used to investigate two research questions:

1. *Profile.* How are the various ways in which students generalize their learning experiences across a range of instructional treatments related conceptually to the various focusing phenomena that get established in the classrooms?

2. *Content and character of generalization.* What is the mathematical content and the character of the generalizations students construct about linear and quadratic functions? What are the trajectories as students develop more sophisticated, powerful generalizations over time?

In order to accomplish these goals we extended our previous work on focusing phenomena from a single case study to the cross-case analyses of multiple instructional units (four in Year 1; eight in Year 2; five in Year 3). By designing the study so that a range of focusing phenomena emerged across the instructional units and by controlling for students' entering content knowledge, the resulting profile reveals how the character of different focusing phenomena are conceptually related to the content and character of students' generalizations. We rely on models of process causality (Maxwell, 2004) and use randomization as a guard against the systemic selection bias that would weaken our ability to make claims about focusing phenomena. Although the number of groups is relatively small, we believe this is a necessary tradeoff given the ambitious effort of engaging in deep qualitative analysis of each of the instructional units.

c. Setting, population, and intervention (if applicable);

Our study occurred in three phases at the Wisconsin site. In Phase 1, we conducted a classroom focusing phenomena study in a high-school algebra II class. Normal classroom instruction was videotaped and transcribed during a unit on quadratic functions in a regular

high-school classroom (with approximately 30 students). The teacher was selected as someone who used a reform textbook and was interested in the use of real world applications and the development of meaningful concepts through student-centered activities. Eight students from the classroom were selected for individual interviews on quadratic functions problems. We conducted two hour-long interviews with each student, and all of the interviews were videotaped and transcribed.

In Phase 2 we tested a small-scale teaching experiment with eight middle school students on quadratic functions. We collected pre- and post-session assessment data on each student in order to track their conceptual change during the teaching experiment. The teaching experiment lasted 15 days and all sessions and interviews were videotaped and transcribed.

Building on the findings from Phase 2, we developed an intervention unit on quadratic functions that we implemented in Phase 3. We randomly assigned eighth-grade student participants to either the intervention unit or the control unit (a typical textbook unit on quadratic functions taught by a middle school teacher). We collected videotaped data on both the control and intervention units, as well as conducting pre- and post-unit interviewed assessments for all participants, which were also videotaped and transcribed.

d. Bullets for the IES topic area(s) and research goal(s) that the project most closely aligns with; the funding agency, amount, and performance period; and the types of roles fellows may have.

- IES Topic: Math-Science
- IES Goal(s): 1, 2,
- Agency: NSF Research on Learning and Education (RoLE). Award: $577,468 (subcontract to UW).
- Performance Period: 9/1/05–8/31/09
- Postdoctoral Fellows Roles: Post-hoc analysis opportunities with extant data sets
    - Open and axial coding techniques
    - Classroom data analysis
    - Cross-case analysis methods

4.a. Project Title & Mentors: **Does visual scaffolding facilitate mathematics learning? Evidence from early algebra. Alibali (PI), Nathan (co-PI), Knuth (co-PI).**

b. Brief description of the research questions and design and analytic methods: The proposed research has three aims: (1) to document how teachers use gestures in naturalistic instructional communication in the domain of early algebra; (2) to investigate whether teachers' gestures promote students' comprehension of instructional language, and therefore their learning; and (3) to investigate a possible mechanism by which gestures promote comprehension and learning—namely, by facilitating students' encoding of visual information. These issues will be examined in the domain of early algebra in middle school.

To address **Aim 1**, we will gather two types of data. In **Study 1**, we will videotape early algebra lessons and analyze teachers' gestures in whole-class and small-group activities. In **Study 2**, we will videotape one-on-one tutorial sessions in which teachers tutor students on a complex mathematical task (symbolizing configurations of a pan balance using algebraic sentences) and analyze teachers' gestures in these tutorial sessions. These studies will document variability in gesture production across teachers, and test the hypothesis that teachers use gestures to scaffold students' comprehension. We will also examine teachers' use of gestures to highlight links among different mathematical representations.

To address **Aim 2**, we will conduct two naturalistic experiments. In both, teachers will be asked to vary their gestures in the context of two lessons (classroom lessons in **Study 3** and tutorial sessions in **Study 4**), such that in one lesson they gesture as they ordinarily do, and in the other, they use additional gestural scaffolding. We will examine whether teachers are able to vary their gestures in the intended ways, and if so, whether students' learning varies as a function of the teachers' gestures. To assess student learning, we will perform detailed analyses of students' performance both before and after the lessons, and we will conduct in-depth interviews with a subset of students. We hypothesize that students will learn more from the lessons that include enhanced gestural scaffolding.

To address **Aim 3**, we will investigate two possible ways in which teachers' gestures may facilitate students' comprehension and learning. The *heightened-attention* hypothesis holds that teachers' gestures promote attentiveness to the lesson as a whole; therefore, gestures should facilitate students' encoding of lesson content *in general*. The *specific-content* hypothesis holds that teachers' gestures promote students' encoding of the referents of those gestures; thus, they should facilitate students' encoding of *specific information.* To distinguish between these hypotheses, **Study 5** will utilize videotaped lessons that identify relations among tables, graphs, and equations. In one lesson, the teacher will highlight only aspects of the table in gestures; in another, she will highlight aspects of the graph. In a third, she will use rhythmic (beat) gestures that are not directed to any of the external representations, and in a fourth, she will use no gestures at all. Students will be randomly assigned to view one of the lessons. Following the lesson, students' encoding of the table, graph, and equation will be tested, and their learning about links between representations will be assessed. If gestures do not facilitate encoding, then students' encoding of all three external representations (table, graph, equation) should be comparable across conditions. If the heightened-attention hypothesis is correct, students' encoding of all three external representations should be better in the three gesture lessons than in the no-gesture lesson. If the specific-content hypothesis is correct, encoding of the table should be best in the highlight-table lesson, encoding of the graph should be best in the highlight-graph lesson, and encoding of the equation should not differ across lessons. We will also examine how variations in students' encoding relate to their learning about links between the representations.

c. Setting, population, and intervention (if applicable): Participants for the studies will be middle school teachers and students from grades 6 and 7. Students will be recruited from a public middle school with an ethnically diverse student population (62% Caucasian, 25% African American, 7% Asian American, and 5% of Hispanic ethnicity) with a wide range of SES. Every effort will be made to have the final sample of students for each study reflect the ethnic and racial diversity of the greater Madison area (82% Caucasian, 6.5% African-American, 6% Asian, 4% Hispanic, & 1.5% other races, according to 2000 Census data). Roughly equal numbers of girls and boys will be included in each study. Teachers will be recruited from the same middle school as the students, and also through professional development programs. For studies that involve teachers, the final sample will reflect the diversity of the pool of potential participants. Every effort will be made to recruit a sample of teachers that is diverse in terms of age, gender, and ethnicity.

Study 1 will involve videotapes of regular classroom lessons in the domain of early algebra. Eight teachers and their students will participate.

In Study 2, teachers will tutor students individually about an early algebra task. Eight teachers and 32 students will participate.

In Study 3, middle school teachers will vary their gestural behavior in pairs of lessons in the domain of early algebra. Following the lessons, their students will complete paper and-pencil

assessments so that their learning can be evaluated. A subset of students will also be interviewed in a one-on-one setting with items similar to those on the paper-and-pencil assessment. A total of six teachers and 240 students will participate. Twenty-four of these students will also participate in the videotaped one-on-one interviews.

In Study 4, middle school teachers will vary their gestural behavior in pairs of tutoring sessions in the domain of early algebra. Following the sessions, students will complete paper-and-pencil assessments to assess learning. A subset of students will be interviewed in a one-on-one setting with items similar to those on the paper-and-pencil assessment. A total of six teachers and 48 students will participate. Twelve of these students will participate in the videotaped one-on-one interviews.

In Study 5, middle school students will view one of four videotaped lessons about an early algebra task. The teachers' gesture will vary across the lessons. Following the lesson, students will be asked to complete a paper-and- pencil assessment, A subset of students will be interviewed in a one-on-one setting with items similar to those on the paper-and-pencil assessment. A total of 120 students will participate, and 12 of these students will participate in the videotaped one-on-one interviews.

d. Bullets for the IES topic area(s) and research goal(s) that the project most closely aligns with; the funding agency, amount, and performance period; and the types of roles fellows may have.

- IES Topic: Cognition and Student Learning
- IES Goal(s): 1, 2.
- Agency: IES. $982,736
- Performance Period: Sept. 2006 – Aug. 2009; No Cost Extension through Aug. 2011.
- Roles for Fellows include:
  - Regression analysis of student performance data
  - Gesture analysis of teacher instruction from video
  - Gesture analysis of student problem solving from video
  - Discourse analysis from video

5.a. Project Title & Mentors: Development of conditional probability judgments. Kalish (PI).

b. Brief description of the research questions and design and analytic methods: The general hypothesis motivating this project is that categorization and projection are best understood in terms of inductive inference (often characterized as a "Bayesian" approach). On inductive accounts, categorization and property projection involve judgments about populations based on samples (e.g., exemplars encountered). The evidential significance of a sample depends on (1) which population one is concerned about (e.g., the conditional one is assessing), and (2) the population sampled from (e.g., how the exemplars were generated). The empirical studies involve small-scale experiments focused on children's learning of conditional probability relations. Experimental manipulations involve the nature of examples children are provided during practice, the forms of feedback they receive, and the types of responses they are asked to make. The data are analyzed quantitatively, with a combination of descriptive and inferential statistics.

c. Setting, population, and intervention (if applicable): The research conducted in a laboratory setting with samples of preschool- and young school-aged children recruited from the Madison Metropolitan area.

d. Bullets for the IES topic area(s) and research goal(s) that the project most closely aligns with; the funding agency, amount, and performance period; and the types of roles fellows may have.

- IES Topic: CASL
- IES Goal(s): 1, 5
- Agency: NSF DLS/DRM. Award: $300,000.
- Performance Period: 9/2008–7/2013
- Postdoctoral Fellows Roles:
    - Experimental design
    - Cognitive modeling
    - Statistical data analysis

6.a. Project Title & Mentors: Understanding and cultivating the connections between students' natural ways of reasoning and mathematical ways of reasoning (IDIOM). Knuth (PI), Ellis (co-PI), Kalish (co-PI).

b. Brief description of the research questions and design and analytic methods: Mathematics education research continues to paint a bleak picture of students' abilities to reason mathematically, yet, in contrast, cognitive science research has revealed surprising strengths in children's abilities to reason in non-mathematical contexts. This raises something of a paradox: Why are children so good at reasoning in non-mathematical contexts, yet so poor at reasoning in mathematical contexts? This project explores this seeming paradox by extending the cognitive science research into the domain of mathematics education, and seeks to understand the relationship between students' reasoning abilities in non-mathematical and mathematical contexts.

The research has two interconnected phases and includes the collection of written survey and interview data. The goal of the Phase I studies is to determine students' representations of mathematical objects. A series of paper-and-pencil surveys and individual interviews were used to determine students' representations of categories and similarity (the basis of inductive strategies in non-mathematical domains) in two content domains within mathematics: number and geometry. One question is whether students have consistent and robust intuitions about similarities between mathematical objects and, critically, what determines those similarities. The goal of the Phase II studies is to determine connections between out-of-mathematics and in-mathematics reasoning and investigate inductive inference in different contexts. The Phase II studies explored how students make inductive inferences, and included questions that asked students to evaluate and generate example-based justifications.

c. Setting, population, and intervention (if applicable): The research setting is middle schools and involves approximately 350 middle school students (grades 6–8).

d. Bullets for the IES topic area(s) and research goal(s) that the project most closely aligns with; the funding agency, amount, and performance period; and the types of roles fellows may have.

- IES Topic: CASL, MSE
- IES Goal: 1
- Agency: NSF REESE Program. Award: $741,938.
- Performance Period: 9/2008–8/2013)
- Postdoctoral Fellows Roles:
    - Analyses of survey and interview data
    - Dissemination of results

7.a. Project Title & Mentors: Developing algebra-ready students for middle school: Exploring the impact of early algebra (LEAP). Knuth (PI)

b. Brief description of the research questions and design and analytic methods: The project seeks to build the essential preliminary components necessary to investigate the impact of algebra in the elementary grades (hereinafter, *early algebra*) on students' algebra readiness in middle grades, and to test an instructional intervention. The goals of the project are:

- To coordinate (empirical) research, curricular, and mathematical perspectives to design a curricular learning progression (CLP) that identifies core algebraic concepts and their progression in children's thinking across upper elementary and middle grades (grades 3–7).
- Using the CLP, to design grade-based assessments of students' algebra understanding for upper elementary and middle grades (grades 3–7) and to validate these assessments through psychometric testing.
- To conduct a preliminary efficacy study concerning the impact of an early algebra intervention (in grades 3–5) based on the CLP and measured by the assessments developed in Project Goal 2.

The CLP was implemented (by a member of the project team) in grades 3–5 at the treatment school site and a second school served as the control site; students at both sites completed grade-based pre- and postassessments (for assessment validation purposes and as part of the efficacy study). Students in grades 6–7 completed a grade-based assessment (for assessment validation purposes).

c. Setting, population, and intervention (if applicable): The research settings are elementary schools (grades 3–5) and middle schools (grades 6–7). In the former case, approximately 300 students participated, and in the latter case, approximately 300 middle school students participated. The early algebra intervention was based on the CLP and consisted of approximately 20 lessons implemented throughout the course of the school year (one lesson a week).

d. Bullets for the IES topic area(s) and research goal(s) that the project most closely aligns with; the funding agency, amount, and performance period; and the types of roles fellows may have.

- IES Topic: CASL, MSE
- IES Goals: 2
- Agency: NSF Discovery Research K–12 (DRK–12) Program. Award: $1,578,658.
- Performance Period: 9/2009–8/2013
- Postdoctoral Fellows Roles:
    - Analyses of assessment data (both quantitative and qualitative analyses)
    - Analysis of teaching intervention data
    - Design of instructional activities
    - Dissemination of results